

# EXCITED HEAVY BARYON MASSES FROM THE $1/N_c$ EXPANSION OF HQET

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The mass spectra of the  $L = 1$  orbitally excited heavy baryons with light quarks in both the spin-flavor symmetric and the mixed representations are studied by the  $1/N_c$  expansion method in the framework of the heavy quark effective theory. The mixing effect between the baryons in the two representations is also considered. The general pattern of the spectrum is predicted which will be verified by the experiments in the near future.

## 1 Introduction

Experimentally, a lot of data for orbitally excited heavy baryons have been accumulating<sup>1</sup>. The following charmed baryon states have been found,  $\Lambda_c(2593)^+$  with  $I(J^P) = 0(\frac{1}{2}^-)$  which is denoted as  $\Lambda_{c1}(\frac{1}{2}^-)^+$ ,  $\Lambda_c(2625)^+$  with  $I(J^P) = 0(?)$  denoted as  $\Lambda_{c1}(\frac{3}{2}^-)^+$ , where  $?$  is  $\frac{3}{2}^-$  from the quark model, and its strange analogues  $\Xi_{c1}(\frac{3}{2}^-)$ .

Theoretical understanding of these baryons is necessary. The heavy quark effective theory (HQET)<sup>2</sup> provides a systematic way to investigate hadrons containing a single heavy quark. To obtain detailed prediction, however, some non-perturbative QCD methods have to be used, such as lattice simulation,  $1/N_c$  expansion<sup>3,4</sup>, chiral Lagrangian and QCD sum rules. In this talk we report the application of the  $1/N_c$  expansion method<sup>5,6</sup>.

Let us first make a brief review of the HQET<sup>2</sup>. It is an effective field theory of QCD for heavy hadrons. In the limit  $m_Q/\Lambda_{\text{QCD}} \rightarrow \infty$ , the heavy quark spin-flavor symmetry (HQS) is explicit. The 4-velocity  $v$  of the heavy quark becomes a good quantum number. Because  $m_Q$  is unusable, it can be

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removed by redefining the heavy quark field:

$$h_v = e^{im_Q v \cdot x} Q . \quad (1)$$

The effective Lagrangian is then

$$\mathcal{L}_{\text{eff}} = \bar{h}_v v \cdot D h_v + O(1/m_Q) . \quad (2)$$

The hadron mass is expanded as

$$\bar{\Lambda}_H + O(1/m_Q) \quad (3)$$

in the effective theory, that is

$$M_H = m_Q + \bar{\Lambda}_H + O(1/m_Q) . \quad (4)$$

Second, let us come to the  $1/N_c$  expansion<sup>3</sup>. This is a non-perturbative method of QCD. The idea is to extract out the non-perturbative information of  $SU(N_c)$  gauge theory by taking  $N_c \rightarrow \infty$ . The  $N_c$  counting rules are given as follows. The interaction vertex is  $g_s/\sqrt{N_c}$ ; the quark propagator keeps unchanged; and the gluon propagator is represented by double lines, one for quarks and the other for anti-quarks.

For the mesons, the non-perturbative properties can be observed from the analysis of the planar diagrams. The large  $N_c$  limit is quite successful. Because the meson decay amplitude  $\sim 1/\sqrt{N_c}$ , mesons and glue states are free and stable. This agrees qualitatively with color confinement. Another example is the explanation of the Zweig's rule.

For the baryons, the diagrammatic method does not work. The Hartree approximation can be adopted. The observation is that in the  $N_c \rightarrow \infty$  limit, interaction between any pair of quarks is negligible ( $\sim 1/N_c$ ); the total potential on an individual quark, which is  $\sim 1$ , is a sum of many small terms, therefore it can be regarded as the background potential or a c-number potential. For ground state baryons, the many-body wave function is written as

$$\Psi(x_1, \dots, x_{N_c}, t) = \sum_1^{N_c} \phi(x, t) , \quad (5)$$

where  $\phi(x, t)$  is one-body state. Interesting results for large  $N_c$  baryons can be obtained. The baryon-(anti-)baryon interaction is order  $N_c$ . And the baryon-meson interaction is order 1. However, the Hartree potential is not known, in which the two-body, three-body, and many-body interactions are the same important. One conjecture is that baryons are solitons of the mesonic theory, skyremions<sup>7</sup>!

Something more can be said about the large  $N_c$  baryons<sup>4</sup>. For the ground state baryons, there is a contracted  $SU(2N_f)$  light quark spin-flavor symmetry (LQS) in the large  $N_c$  limit. This was first obtained from the chiral perturbation theory of baryon-pion interactions in deriving the consistent conditions for the coupling constants in the large  $N_c$  limit. It can be also understood in the Hartree picture. This makes a  $1/N_c$  expansion based on the spin-flavor structure practically possible for the baryons. Many quantitative predictions and further extensions of the above result have been made<sup>8</sup>. In fact, this expansion is another scheme of the  $1/N_c$  expansion. This can be simply seen from considering the masses of the non-strange baryons,

$$\begin{aligned} M_H &= N_c \Lambda_{\text{QCD}} + O(1) + O\left(\frac{1}{N_c}\right) + \dots \\ &= N_c \tilde{\Lambda}_{\text{QCD}} + c_1 \frac{S^2}{N_c} + c_2 \frac{(S^2)^2}{N_c^2} + \dots, \end{aligned} \quad (6)$$

where the first line is the ordinary  $1/N_c$  expansion, and the second one the expansion based on the spin-flavor structure. Of course, in the  $N_c \rightarrow \infty$  limit,  $M_H = \tilde{\Lambda}_H = N_c \Lambda_{\text{QCD}} = N_c \tilde{\Lambda}_{\text{QCD}} = m_{\text{proton}}$  which is not so useful.

## 2 Excited Heavy Baryons in the $1/N_c$ Expansion

For the charmed baryons like  $\Lambda_{c1}(\frac{1}{2}^-)^+$ ,  $\Lambda_{c1}(\frac{3}{2}^-)^+$ ,  $\Xi_{c1}(\frac{3}{2}^-)^+$ , we analyze their masses  $\bar{\Lambda}_H$  in the  $1/N_c$  expansion. The classification of them is according to the angular momentum  $J$ , the isospin  $I$  and the total angular momentum of the light degrees of freedom  $J^l$  which becomes a good quantum number due to HQS. In this case, the excited hadron spectrum shows the degeneracy of pair of states which are related to each other by HQS. Constituently there are two ways for the  $L = 1$  excitation. One is that the heavy quark is excited; the other is that one light quark is excited. Correspondingly under the LQS, the  $N_c - 1$  light quarks are in the symmetric and mixed representation, respectively.

### 2.1 Symmetric Representation

In the symmetric representation, the picture for the light quarks is essentially the same as that of the ground state heavy baryons. The spin-flavor decomposition rule is  $I = S^l$  for the non-strange baryons, where  $S^l$  is the total spin of the light quark system. Note that the light quark system as a whole has  $L = 1$  orbital angular momentum. All possible states of excited heavy baryons are listed in Table 1.

In the Hartree–Fock picture of the baryons, the  $N_c$  counting rules require us to include many-body interactions in the analysis. However, a large part of

Table 1: Excited heavy baryon states of the symmetric representation of  $N_c - 1$  light quarks.

$(J, I)$	$(J^I, S^I)$	$\bar{\Lambda}_H^0$
(1/2, 0)	(1, 0)	$N_c c_0 + 2c_1$
(3/2, 0)	(1, 0)	$N_c c_0 + 2c_1$
(1/2, 1)	(0, 1)	$N_c c_0 - 2c_1 + \frac{2c_2}{N_c}$
(1/2, 1)	(1, 1)	$N_c c_0 + \frac{2c_2}{N_c}$
(3/2, 1)	(1, 1)	$N_c c_0 + \frac{2c_2}{N_c}$
(3/2, 1)	(2, 1)	$N_c c_0 + 4c_1 + \frac{2c_2}{N_c}$
(5/2, 1)	(2, 1)	$N_c c_0 + 4c_1 + \frac{2c_2}{N_c}$

these interactions are spin-flavor irrelevant. Namely this part contributes in the order  $N_c \Lambda_{\text{QCD}}$  universally to all the baryons with different spin-flavor structure in Table 1. The mass splittings among the baryons can be obtained. For the purely light quark contribution to  $\bar{\Lambda}_H$ , the  $1/N_c$  analysis goes the same as that to the ground state heavy baryons. There is LQS at the leading order of the  $1/N_c$  expansion. The mass splitting due to the violation of LQS started from  $S^{I^2}/N_c$ . However, different from the ground state baryons, formally the orbital angular momentum of the heavy quark has more dominant contribution to  $\bar{\Lambda}_H$  than  $O(1/N_c)$ . This is because of the orbital-light-quark-spin interactions. After summing up all the relevant many-body interactions, this order  $O(1)$  contribution is  $\vec{L} \cdot \vec{S}^I f(\frac{S^{I^2}}{N_c^2})$ , where  $f$  is a general function which can be Taylor expanded. The mass  $\bar{\Lambda}_H$  can be written simply as

$$\bar{\Lambda}_H^0 = N_c \tilde{c}_0 + \tilde{c}_1 \vec{L} \cdot \vec{S}^I + O\left(\frac{1}{N_c}\right), \quad (7)$$

where coefficients  $\tilde{c}_i \sim \Lambda_{\text{QCD}}$  ( $i = 0, 1$ ). There should be also a term proportional to  $L^2$  in the above equation, which gives constant contribution to  $\bar{\Lambda}_H^0$  for a given light quark representation, and therefore has been absorbed into the leading term. The term  $\vec{L} \cdot \vec{S}^I$  can be rewritten as  $J^{I^2} - S^{I^2}$  with  $\vec{J}^I$  being defined as  $\vec{J}^I = \vec{L} + \vec{S}^I$ . Therefore

$$\bar{\Lambda}_H^0 = N_c c_0 + c_1 (J^{I^2} - S^{I^2}) + O\left(\frac{1}{N_c}\right), \quad (8)$$

where coefficients  $c_i \sim \Lambda_{\text{QCD}}$ .

Table 2: Excited heavy baryon states of the mixed representation of  $N_c - 1$  light quarks.

$(J, I)$	$(J^I, S^I)$	$\Lambda_{H'}^0$
$(1/2, 0)$	$(0, 1)$	$-2c_{LS} + \frac{1}{18}c_T - \frac{1}{3}\bar{c}_1 + \frac{1}{2}\bar{c}_2$
$(1/2, 0)$	$(1, 1)$	$-c_{LS} + \frac{1}{18}c_T - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2$
$(3/2, 0)$	$(1, 1)$	$-c_{LS} + \frac{1}{18}c_T - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2$
$(3/2, 0)$	$(2, 1)$	$9c_{LS} + \frac{1}{18}c_T + \frac{1}{6}\bar{c}_1 + \frac{1}{20}\bar{c}_2$
$(5/2, 0)$	$(2, 1)$	$9c_{LS} + \frac{1}{18}c_T + \frac{1}{6}\bar{c}_1 + \frac{1}{20}\bar{c}_2$
$(1/2, 1)$	$(1, 0)$	$\frac{1}{18}c_T$
$(3/2, 1)$	$(1, 0)$	$\frac{1}{18}c_T$

## 2.2 Mixed Representation

In the mixed representation, all states are listed in Table 2. Again,  $\bar{\Lambda}_{H'}$  is trivially  $N_c \Lambda_{\text{QCD}}$  at the leading order of the  $1/N_c$  expansion. The spin-flavor dependence, however is more complicated. For the spectrum of excited light baryons, see ref. <sup>8</sup>.

The many-body Hamiltonians related to the spin-flavor structure which involve orbital angular momentum  $L$  give  $O(1)$  contribution. We use the following operators which were used in <sup>8</sup> to analyze  $\bar{\Lambda}_H$ ,

$$\begin{aligned}
H_{LS} &\propto \hat{a}^\dagger \vec{L} \cdot \vec{\sigma} \hat{a} \\
H_T &\propto \frac{1}{N_c} G^{ia} G_{ia} \\
H_1 &\propto \frac{1}{N_c} \hat{a}^\dagger L^i \otimes \tau^a \hat{a} G_{ia} \\
H_2 &\propto \frac{1}{N_c} \hat{a}^\dagger \{L^i, L^j\} \otimes \sigma_i \otimes \tau^a \hat{a} G_{ja} .
\end{aligned} \tag{9}$$

The first one  $H_{LS}$  is one-body Hamiltonian, while the others are two-body Hamiltonians.  $G_{ia}$  are the generators of the spin-flavor symmetry group  $\text{SU}(4)$ , given by

$$G^{ia} = \hat{a}^\dagger \sigma^i \otimes \tau^a \hat{a} \tag{10}$$

with  $\sigma^i$  and  $\tau^a$  being the spin and isospin matrices, respectively. Such structure gives coherent addition over  $N_c - 2$  core quarks. The first  $G^{ia}$  in  $H_T$  acts on the excited quark, the other  $G_{ia}$ 's on the  $N_c - 2$  unexcited light quarks, namely the core quarks. In our case, all the operators must be understood as the ones acting on the light degrees of freedom. Note that the higher order many-body Hamiltonian which contains more factor of  $G_{ia}$  can be reduced to those given in Eq.(9).

The contributions to the baryon masses due to these Hamiltonians are obtained by calculating the baryonic matrix elements. The matrix elements of these operators between the states of light quarks which specify the states of excited heavy baryons are given as follows,

$$\begin{aligned}
& \langle I_c = \frac{1}{2}; I I_3; S^\nu S_3^\nu, l = 1 m' | H_T | I_c = \frac{1}{2}; I I_3; S^l S_3^l, l = 1 m \rangle \\
& = 2c_T \delta_{S^\nu, S^l} \delta_{S_3^\nu, S_3^l} \delta_{m, m'} (-1)^{1-S^l-I} \left\{ \begin{matrix} S^l & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & \frac{1}{2} \end{matrix} \right\} \left\{ \begin{matrix} \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & I & \frac{1}{2} \end{matrix} \right\} ,
\end{aligned} \tag{11}$$

$$\begin{aligned}
& \langle I_c = \frac{1}{2}; I I_3; l = 1, S^\nu, J^l J_3^l | H_{LS} | I_c = \frac{1}{2}; I I_3; l = 1, S^l, J^l J_3^l \rangle \\
& = c_{LS} (-1)^{S^l-S^\nu} \sqrt{(2S^l+1)(2S^\nu+1)} \sum_{j=\frac{1}{2}, \frac{3}{2}} (2j+1) \{j(j+1) - 2 - 3/4\} \\
& \quad \left\{ \begin{matrix} \frac{1}{2} & \frac{1}{2} & S^l \\ 1 & J^l & j \end{matrix} \right\} \left\{ \begin{matrix} \frac{1}{2} & \frac{1}{2} & S^\nu \\ 1 & J^l & j \end{matrix} \right\} ,
\end{aligned} \tag{12}$$

$$\begin{aligned}
& \langle I_c = \frac{1}{2}; I I_3; l = 1, S^\nu, J^l J_3^l | H_1 | I_c = \frac{1}{2}; I I_3; l = 1, S^l, J^l J_3^l \rangle \\
& = 6\bar{c}_1 (-1)^{I-J^l+S^l-S^\nu-1} \sqrt{(2S^l+1)(2S^\nu+1)} \left\{ \begin{matrix} \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & I & \frac{1}{2} \end{matrix} \right\} \left\{ \begin{matrix} S^l & 1 & S^\nu \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{matrix} \right\} \left\{ \begin{matrix} 1 & 1 & 1 \\ S^\nu & J^l & S^l \end{matrix} \right\} ,
\end{aligned} \tag{13}$$

$$\begin{aligned}
& \langle I_c = \frac{1}{2}; I I_3; l = 1, S^\nu, J^l J_3^l | H_2 | I_c = \frac{1}{2}; I I_3; l = 1, S^l, J^l J_3^l \rangle \\
& = 3\bar{c}_2 (-1)^{1+J^l+I+S^\nu+2S^l} \sqrt{(2S^l+1)(2S^\nu+1)} \left\{ \begin{matrix} \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & I & \frac{1}{2} \end{matrix} \right\} \left\{ \begin{matrix} 2 & 1 & 1 \\ J^l & S^\nu & S^l \end{matrix} \right\} \left\{ \begin{matrix} S^\nu & S^l & 2 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{matrix} \right\} 14)
\end{aligned}$$

where  $I_c$  is the isospin of the core quarks. In real world ( $N_c = 3$ ), there is only one quark in the core so  $I_c$  always equals  $\frac{1}{2}$ . With these matrix elements, we can express the excited heavy baryon mass up to the zeroth order of  $1/N_c$ :

$$\bar{\Lambda}_{H'} = N_c \bar{c}_0 + \langle H_{LS} \rangle + \langle H_T \rangle + \sum_{i=1}^2 \langle H_i \rangle , \tag{15}$$

where  $c_{LS}$ ,  $c_T$ , and  $\bar{c}_i$ 's are the coefficients  $\sim \Lambda_{\text{QCD}}$ .

### 2.3 Mixing

It is necessary to consider the mixing between the baryons with light quarks in the spin-flavor symmetric and mixed representations. When they have same good quantum numbers of  $(J, I, J^I)$ , there is no physical way to distinguish them. This consideration will give the physical spectrum. Because of the light quark spin-flavor symmetry at the leading order of  $1/N_c$  expansion, the baryons with same  $(J, I, J^I)$  quantum numbers but in different representations do not mix. ( $S^I$  is a good quantum number in  $N_c \rightarrow \infty$ .)<sup>9</sup> The mixing occurs at the sub-leading order. The classification of baryons by the spin-flavor symmetry is therefore physical at the leading order. For the physical spectrum, the mixing results in a deviation from  $\bar{\Lambda}_H^0$ . By denoting the mixing mass as  $\tilde{m}$  which is of  $O(1)$ , the mass matrix for the baryons with same  $(J, I, J^I)$  is written as

$$\begin{pmatrix} \bar{\Lambda}_H^0 & \tilde{m} \\ \tilde{m} & \bar{\Lambda}_{H'}^0 \end{pmatrix}. \quad (16)$$

The mass difference  $\bar{\Lambda}_H^0 - \bar{\Lambda}_{H'}^0$  is  $O(1)$ . Taking  $\tilde{m} < \bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0$  for illustration, the physical masses are corrected to be

$$\begin{aligned} \bar{\Lambda}_H &\simeq \bar{\Lambda}_H^0 - \frac{\tilde{m}^2}{\bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0}, \\ \bar{\Lambda}_{H'} &\simeq \bar{\Lambda}_{H'}^0 + \frac{\tilde{m}^2}{\bar{\Lambda}_{H'}^0 - \bar{\Lambda}_H^0}. \end{aligned} \quad (17)$$

The  $1/N_c$  expansion of  $\tilde{m}$  is parameterized as

$$\tilde{m} = \tilde{m}_0 + O(1/N_c), \quad (18)$$

where  $\tilde{m}_0$  is universal due to LQS. To the order of  $O(1)$ , the spectrum is,

$$\begin{aligned} \bar{\Lambda}_{(\frac{1}{2}(\frac{3}{2}),0,1)} &= N_c c_0 + 2c_1 - \frac{\tilde{m}_0^2}{k - c_{LS} - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2 - 2c_1}, \\ \bar{\Lambda}_{(\frac{1}{2},1,0)} &= N_c c_0 - 2c_1, \\ \bar{\Lambda}_{(\frac{1}{2}(\frac{3}{2}),1,1)} &= N_c c_0 - \frac{\tilde{m}_0^2}{k}, \\ \bar{\Lambda}_{(\frac{3}{2}(\frac{5}{2}),1,2)} &= N_c c_0 + 4c_1, \end{aligned} \quad (19)$$

where  $k$  is an  $O(1)$  constant which is what remains after the  $\bar{\Lambda}_{H'}^0$  and  $\bar{\Lambda}_H^0$  cancellation. Note that the masses of the states  $(\frac{1}{2}, 1, 0)$  and  $(\frac{3}{2}(\frac{5}{2}), 1, 2)$  are

not affected by the mixing, because there are no physical states with the same good quantum numbers in the mixed representation. From the above spectrum, we see that  $c_1 > 0$ . The states  $(\frac{3}{2}(\frac{5}{2}), 1, 2)$  is always the highest states. They are heavier than the other states at least by  $4c_1$  through requiring the states  $(\frac{1}{2}(\frac{3}{2}), 0, 1)$  to be the lowest. If  $2c_1 > \frac{\tilde{m}_0^2}{k}$ , the requirement implies

$$\frac{\tilde{m}_0^2}{k - c_{LS} - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2 - 2c_1} > 4c_1 . \quad (20)$$

In this case, the spectrum pattern is

$$M(\frac{1}{2}(\frac{3}{2}), 0, 1) < M(\frac{1}{2}, 1, 0) < M(\frac{1}{2}(\frac{3}{2}), 1, 1) < M(\frac{3}{2}(\frac{5}{2}), 1, 2) . \quad (21)$$

On the other hand, if  $2c_1 < \frac{\tilde{m}_0^2}{k}$ , the requirement is

$$\tilde{m}_0^2 \left( \frac{1}{k - c_{LS} - \frac{1}{6}\bar{c}_1 - \frac{1}{4}\bar{c}_2 - 2c_1} - \frac{1}{k} \right) > 2c_1 , \quad (22)$$

which gives the spectrum

$$M(\frac{1}{2}(\frac{3}{2}), 0, 1) < M(\frac{1}{2}(\frac{3}{2}), 1, 1) < M(\frac{1}{2}, 1, 0) < M(\frac{3}{2}(\frac{5}{2}), 1, 2) . \quad (23)$$

The experimentally found baryons  $\Lambda_{c1}(\frac{1}{2})$  and  $\Lambda_{c1}(\frac{3}{2})$  correspond to the  $(\frac{1}{2}(\frac{3}{2}), 0, 1)$  states. More data are needed to fix the unknown parameters  $c_i$ 's,  $\bar{c}_i$ 's,  $k$  and  $c_{LS}$ . In the near future, experiments will check the above predicted spectrum. Hopefully one of the above mass patterns will be picked out. It will be a check for the validity of our method, if the parameters are in the reasonable range ( $\Lambda_{\text{QCD}}$ ) and meanwhile satisfy the relations given above.

### 3 Summary

In summary, we have reported the  $1/N_c$  expansion method in studying the spectra of the  $L = 1$  orbitally excited heavy baryons within the framework of HQET. The analysis is very simple for the baryons with light quarks being in the spin-flavor symmetric representation, compared to that for the heavy baryons with light quarks in the mixed representation. The simplicity is an unique feature. It can be seen from the point that the light quark system is in the ground state and it is the heavy quark that is orbitally excited. However



the mixing effect due to the baryon states in the mixed representation corrects the spectrum pattern in the sub-leading order of  $1/N_c$  expansion. The effect is important to get the realistic spectra at this order. The general pattern of the baryon spectrum has been given, which will be verified by the experiments in the near future. The  $1/m_Q$  and SU(3) corrections have been considered in ref. <sup>5</sup>. Certain mass relations for the baryons  $\Lambda_{c1}^{(*)}$ ,  $\Sigma_{c1}^{(*)}$ ,  $\Xi_{c1}^{(')(*)}$ , and  $\Omega_{c1}^{(*)}$  have been derived. The same analysis can be applied to the bottom baryons.

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